Non-conserved currents and gauge-restoring schemes in single W production

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Non-conserved currents and gauge-restoring schemes in single W production

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Abstract

We generalize the inclusion of the imaginary parts of the fermionic one-loop corrections for processes with unstable vector bosons to the case of massive external fermions and non conservation of weak currents. We study the effect of initial and final state fermion masses in single W production in connection with the gauge-invariant treatment of the finite-width effects of W and Z bosons, giving numerical comparisons of different gauge-invariance-preserving schemes in the energy range of LEP2 and LC for $e^- e^+ \rightarrow e^- \bar{\nu}_e u\bar{d}$. We do not find significant differences between the results obtained in the imaginary part fermion loop scheme and in other exactly gauge preserving methods. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

In the past few years a number of papers have discussed the inclusion of weak boson finite-width effects in the theoretical predictions for e^+e^- processes. A careful treatment is required since these effects are intimately related to the gauge invariance of the theory and any violation of Ward identities can lead to large errors. Even recently a new proposal for handling unstable particle processes has appeared [1].

The most appealing approach used in actual numerical computations is in our opinion the Fermion-Loop (FL) scheme [2–5], which consists in the resummation of the fermionic one-loop corrections to the vector-boson propagators and the inclusion of all remaining fermionic one-loop corrections, in particular those to the Yang–Mills vertices. In Ref. [2,3] only the imaginary parts of the loops were included since these represent the minimal set of one–loop contributions which is required for preserving gauge invariance. This scheme will be referred to as the Imaginary Part Fermion-Loop (IFL) scheme in the following. In [5] all contributions from fermionic

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one-loop corrections have been computed. Some effects of light fermion masses in the fermionic loops have been investigated in [6].

In this paper we study the effects of external particle fermion masses which imply the non–conservation of the weak currents which couple to the fermionic loops. These effects have been as yet neglected for $e^+e^- \to 4f$ processes: in Ref. [2] and in the numerical part of Ref. [5] all fermions have been assumed to be massless, while in Ref. [6] massive matrix elements together with the FL corrections of Ref. [5] were used under the assumption that the currents were conserved. Since our main focus is on gauge invariance, we restrict our attention to the imaginary parts of the fermionic loops, generalizing the approach of Ref. [2]. The extension of the full FL scheme to the case of massive external fermions is at present being studied [7] and it will allow to determine the scale of α_{QED} for single W processes.

We compare the different gauge-restoring schemes in $e^-e^+ \to e^-\bar{\nu}_e u\bar{d}$ (CC20) which, in addition to the usual diagrams of $e^-e^+ \to \mu^-\bar{\nu}_\mu u\bar{d}$ (CC10), requires all diagrams obtained exchanging the incoming e^+ with the outgoing e^-. These contributions become dominant for $\theta_e \to 0$ because of the t-channel γ propagator. The CC20 four fermion events with e lost in the pipe are often referred to as single W production, and are relevant for triple gauge studies and as background to searches. For recent reviews see Refs. [9]. Since the t-channel γ propagator diverges at $\theta_e = 0$ in the $m_e \to 0$ limit, fermion masses have to be exactly accounted for. Moreover, the apparent t^{-2} behaviour is reduced to t^{-1} by gauge cancellations. This implies that even a tiny violation of gauge conservation can have dramatic effects, as e.g. discussed in Ref. [2,8], and the use of some gauge conserving scheme is unavoidable.

Two different strategies have been used: Improved Weiszacker-Williams [10] implemented in `WTO` [11] and completely massive codes. In the first case one separates the 4 t-channel photon diagrams, evaluates them analytically in the equivalent photon approximation taking into account the complete dependence on all masses, and then adds the rest of diagrams and the interference between the two sets in the massless approximation. In the fully massive MC numerical approach `COMPHEP` [12], `GRC4F` [13], `KORALW` [14], `WPHACT` [15] and recently also the two new codes `NEXTCALIBUR` [16] and `SWAP` [17] have compared their results and found good agreement [9] [2] among themselves and with `WTO`.

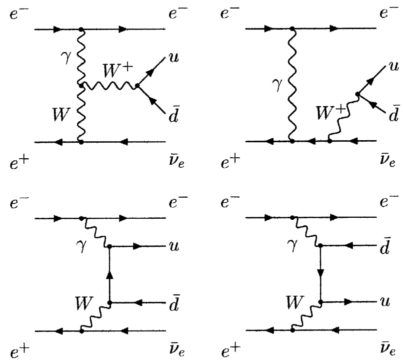

Fig. 1. The four diagrams of the process $e^-(p_1)e^+(k_1) \to e^-(p_2)\bar{\nu}_e(k_2)u(p_u)\bar{d}(p_d)$ which are considered in this paper.

In the following we first discuss the issue of U(1) gauge invariance in single W production with non conserved weak currents. We then give the expression of all required contributions to the vertex corrections in the IFL scheme. Finally we present comparisons between the IFL and other gauge-preserving schemes which have been employed in the literature and study the relevance of neglecting current non conservation in the energy range of LEP2 and LC.

2. Gauge invariance

We choose to work in the unitary gauge. In this case, the relevant set of Feynman diagrams which become dominant for $\theta_e \to 0$ coincides with those discussed in Ref. [2]. They are shown in Fig. 1. For ease of comparison we follow closely the notation of [2]. The corresponding matrix element $\mathscr{M}$ is given by

$$\mathscr{M} = \mathscr{M}^\mu J_\mu, \quad J^\mu = \frac{Q_e}{q^2}\bar{u}(p_2)\gamma^\mu u(p_1),$$

$$\mathscr{M}^\mu = \sum_{i=1}^{4} \mathscr{M}_i^\mu \tag{1}$$

[2] More details can be found in the homepage of the LEP2 MC Workshop `http://www.ph.unito.it/~giampier/lep2.html`

where

$$\mathscr{M}_1^{\mu} = Q_W\, P_W(p_+^2)\, P_W(p_-^2)\, V^{\alpha\beta\mu}(p_+, -p_-, -q) \times D_\alpha^\rho(p_+)\, D_\beta^\sigma(p_-)\, \mathscr{M}_{\sigma\rho}^0 ,$$

$$\mathscr{M}_2^{\mu} = 4iQ_e g_w^2\, P_W(p_+^2)\, \bar{v}(k_1)\gamma^\mu \frac{\not{k}_1 + \not{q} - m_e}{(k_1+q)^2 - m_e^2} \times \gamma^\alpha P_L v(k_2)\, \bar{u}(p_u)\gamma_\rho P_L v(p_d)\, D_\alpha^\rho(p_+) ,$$

$$\mathscr{M}_3^{\mu} = -4iQ_u g_w^2\, P_W(p_-^2)\, \bar{u}(p_u) \times \gamma^\mu \frac{\not{p}_u - \not{q} + m_u}{(p_u - q)^2 - m_u^2} \gamma^\beta P_L v(p_d) \times \bar{v}(k_1)\gamma_\sigma P_L v(k_2)\, D_\beta^\sigma(p_-) ,$$

$$\mathscr{M}_4^{\mu} = -4iQ_d g_w^2\, P_W(p_-^2)\, \bar{u}(p_u) \times \gamma^\beta P_L \frac{\not{q} - \not{p}_d + m_d}{(p_d - q)^2 - m_d^2} \gamma^\mu v(p_d) \times \bar{v}(k_1)\gamma_\sigma P_L v(k_2)\, D_\beta^\sigma(p_-) ,$$

$$\mathscr{M}_{\sigma\rho}^0 \equiv 4ig_w^2\, \bar{v}(k_1)\gamma_\sigma P_L v(k_2)\, \bar{u}(p_u)\gamma_\rho P_L v(p_d) , \quad (2)$$

where $P_L \equiv \frac{1}{2}(1-\gamma^5)$ and

$$p_+ = p_u + p_d ,\; p_- = k_1 - k_2 ,\; q = p_1 - p_2 , \quad (3)$$

$$[P_W(s)]^{-1} = s - M_W^2 + i\gamma_W(s) , \quad (4)$$

$$D_\alpha^\beta(p) = g_\alpha^\beta - p_\alpha p^\beta / K(p^2) . \quad (5)$$

M_W is the W mass and γ_W denotes the imaginary part of the inverse W propagator. At tree level, $K(p^2) = M_W^2$ but the resummation of the imaginary parts of higher order graphs modifies the lowest order expression of K in addition to generate a finite width. The charged weak coupling constant g_w is given by $g_w^2 = M_W^2 G_F/\sqrt{2}$, while Q_i is the electric charge of particle i, and

$$V^{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) = (p_1 - p_2)^{\mu_3} g^{\mu_1\mu_2} + (p_2 - p_3)^{\mu_1} g^{\mu_2\mu_3} + (p_3 - p_1)^{\mu_2} g^{\mu_3\mu_1} . \quad (6)$$

The conservation of electromagnetic current requires

$$q^\mu \mathscr{M}_\mu = 0 . \quad (7)$$

Any small violation of this relation will be amplified by a huge factor and will lead to totally wrong predictions for almost collinear electrons [2,8]. Multiplying q^μ into the four diagrams of Eq. (2), we obtain

$$\begin{aligned} W \equiv q^\mu \mathscr{M}_\mu = \mathscr{M}_0 \{ &(p_+^2 - p_-^2) Q_W\, P_W(p_+^2)\, P_W(p_-^2) \\ &+ Q_e\, P_W(p_+^2) - (Q_d - Q_u)\, P_W(p_-^2) \} \\ &- \mathscr{M}_{++} \{ Q_W\, P_W(p_+^2)\, P_W(p_-^2) \\ &\times (1 - p_-^2/K(p_+^2)) + Q_e\, P_W(p_+^2)/K(p_+^2) \} \\ &+ \mathscr{M}_{--} \{ Q_W\, P_W(p_+^2)\, P_W(p_-^2) \\ &\times (1 - p_+^2/K(p_-^2)) \\ &+ (Q_d - Q_u)\, P_W(p_-^2)/K(p_-^2) \} \\ &+ \mathscr{M}_{-+} \Big\{ Q_W\, P_W(p_+^2)\, P_W(p_-^2) \\ &\times p_- \cdot p_+ \Big(K(p_-^2)^{-1} - K(p_+^2)^{-1} \Big) \Big\} , \end{aligned} \quad (8)$$

where

$$\mathscr{M}_0 \equiv \mathscr{M}_{\alpha\beta}^0\, g^{\alpha\beta} , \quad \mathscr{M}_{++} \equiv \mathscr{M}_{\alpha\beta}^0\, p_+^\alpha p_+^\beta , \quad (9)$$

$$\mathscr{M}_{--} \equiv \mathscr{M}_{\alpha\beta}^0\, p_-^\alpha p_-^\beta , \quad \mathscr{M}_{-+} \equiv \mathscr{M}_{\alpha\beta}^0\, p_-^\alpha p_+^\beta . \quad (10)$$

Using $Q_W = Q_e = Q_d - Q_u = -1$ and Eq. (4) we have

$$\begin{aligned} W = i\, \mathscr{M}_0\, &P_W(p_+^2)\, P_W(p_-^2) (\gamma_W(p_+^2) - \gamma_W(p_-^2)) \\ &+ \mathscr{M}_{++} \{ P_W(p_+^2)\, P_W(p_-^2) \\ &\times (1 - (M_W^2 - i\gamma_W(p_-^2))/K(p_+^2)) \} \\ &- \mathscr{M}_{--} \{ P_W(p_-^2)\, P_W(p_+^2) \\ &\times (1 - (M_W^2 - i\gamma_W(p_+^2))/K(p_-^2)) \} \\ &- \mathscr{M}_{-+} \Big\{ P_W(p_+^2)\, P_W(p_-^2) \\ &\times p_- \cdot p_+ \Big(K(p_-^2)^{-1} - K(p_+^2)^{-1} \Big) \Big\} . \end{aligned} \quad (11)$$

Current conservation is therefore violated unless

$$\gamma_W(p_+^2) = \gamma_W(p_-^2) \equiv \bar{\gamma}_W \quad (12)$$

$$K(p_+^2) = K(p_-^2) = M_W^2 - i\bar{\gamma}_W \quad (13)$$

It should be mentioned that all effects due to the non conservation of the currents which couple to the W and Z bosons are contained in the last three terms of Eq. (8) and Eq. (11) which would be zero if the currents were conserved.

The most naive treatment of a Breit-Wigner resonance uses a *fixed width* approximation, with

$$\bar{\gamma}_W = M_W \Gamma_W . \tag{14}$$

Eqs. (11)–(13) show that in this case there is no violation of electromagnetic current conservation. In the unitary gauge this corresponds to adding the same imaginary part, $-iM_W \Gamma_W$, to M_W^2 both in the denominator and in the $p^\mu p^\nu$ term of the W propagator (see e.g. [3] and references therein). We have verified numerically that neglecting to modify the latter leads to large errors already at 800 GeV. A similar approach, in which all weak boson masses squared $M_B^2, B = W, Z$ are changed to $M_B^2 - i\gamma_B$ everywhere, including in the definition of the weak mixing angle, has in fact been suggested [18] as a mean of preserving both U(1) and SU(2) Ward identities in the Standard Model.

The fixed-width approximation cannot however be justified from field theory. Indeed, propagators with space-like momenta are real and cannot acquire a finite width in contradiction to the fixed-width scheme.

As discussed in Ref. [2], the simplest way to restore gauge-invariance in a theoretically satisfying fashion is the addition of the imaginary parts of one-loop fermionic vertex corrections, shown in Fig. 2, which cancel the imaginary part in the Ward identities. The cancellation is exact as long as all fermion loops, both in the vertices and in the propagators, are computed in the same approximation. In particular we can consistently neglect fermion masses in the loops, if we use for the W width the tree–level expression for the decay of an on-shell W to massless fermions

$$\Gamma_W = \sum_{\text{doublets}} N_f \frac{G_F M_W^3}{6\pi\sqrt{2}} , \tag{15}$$

involving a sum over all fermion doublets with N_f (1 or 3) colours.

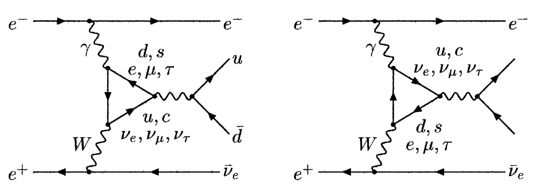

Fig. 2. The extra fermionic diagrams needed to cancel the gauge-breaking terms.

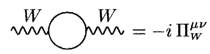

Fig. 3. First order contribution to the inverse W propagator. The fermions in the loop are assumed to be massless.

The vertex corrections are given by

$$\begin{aligned}\mathscr{M}_5^\mu = {} & \frac{i}{16\pi} \mathscr{M}_{\rho\sigma}^0\, P_W(p_+^2)\, P_W(p_-^2)\, g_w^2 \\ & \times \sum_{\text{doublets}} N_f (Q_d - Q_u)\, D_\alpha^\rho(p_+) \\ & \times D_\beta^\sigma(p_-)\, Z^{\alpha\beta\mu} ,\end{aligned} \tag{16}$$

where

$$Z^{\alpha\beta\mu} = \frac{1}{2\pi} \int d\Omega \ \mathrm{Tr}\left[\not{r}_1 \gamma^\mu \frac{\not{r}_1 - \not{q}}{(r_1 - q)^2} \gamma^\beta \not{r}_2 \gamma^\alpha \right] \tag{17}$$

is the imaginary part of the triangle insertions. The momenta r_1 and r_2 are the momenta of the cut fermion lines with $p_+ = r_1 + r_2$. The expression $Z^{\alpha\beta\mu}$ satisfies the three Ward identities:

$$\begin{aligned}& Z^{\alpha\beta\mu} q_\mu = -\tfrac{8}{3}\left(p_+^\alpha p_+^\beta - p_+^2 g^{\alpha\beta} \right), \quad Z^{\alpha\beta\mu} p_\alpha^+ = 0 , \\ & Z^{\alpha\beta\mu} p_\beta^- = +\tfrac{8}{3}\left(p_+^\mu p_+^\alpha - p_+^2 g^{\mu\alpha} \right).\end{aligned} \tag{18}$$

Because of the anomaly cancellation we have no explicit contributions from the part containing γ^5. Attaching the photon momentum q_μ to the sum of the diagrams $\mathscr{M}_5^\mu$ gives

$$\begin{aligned} W_{\text{add}} \equiv q_\mu \mathscr{M}_5^\mu = {} & -i\,\mathscr{M}_0\, P_W(p_+^2)\, P_W(p_-^2)\, \Gamma_W \frac{p_+^2}{M_W} \\ & + i\,\mathscr{M}_{++}\, P_W(p_+^2)\, P_W(p_-^2)\, \frac{\Gamma_W}{M_W} \\ & + i\,\mathscr{M}_{--}\, P_W(p_+^2)\, P_W(p_-^2)\, \frac{\Gamma_W}{M_W} \frac{p_+^2}{K(p_-^2)} \\ & - i\,\mathscr{M}_{-+}\, P_W(p_+^2)\, P_W(p_-^2)\, \frac{\Gamma_W}{M_W} \frac{p_+ \cdot p_-}{K(p_-^2)} \end{aligned} \tag{19}$$

where we used the Ward identity of Eq. (18) and the definition of the nominal W width, Eq. (15). Assuming $\gamma_W(p_-^2)=0$ as required by field theory, the extra diagrams restore U(1) gauge invariance provided

$$\gamma_W(p_+^2)=\Gamma_W\frac{p_+^2}{M_W}, \tag{20}$$

$$K(p_+^2)=M_W^2\left(1+i\frac{\Gamma_W}{M_W}\right)^{-1}, \tag{21}$$

$$K(p_-^2)=M_W^2. \tag{22}$$

This result may be surprising but it is actually the correct field theoretical resummation, in the unitary gauge, of the imaginary part of the fermionic one-loop contributions [3], shown in Fig. 3, which is transverse if we consider only massless fermions

$$\mathrm{Im}(\Pi_W^{\mu\nu})=(g^{\mu\nu}-p^\mu p^\nu/p^2)\Pi_W, \tag{23}$$

with

$$\Pi_W=p^2\frac{\Gamma_W}{M_W}. \tag{24}$$

If, suppressing indices for simplicity, we define $\mathbb{1}\equiv g^{\mu\nu}$, $D\equiv g^{\mu\nu}-p^\mu p^\nu/M^2$ and $T\equiv g^{\mu\nu}-p^\mu p^\nu/p^2$ we have $DT=TD=T$ and $T^2=T$. The usual Dyson series for the resummation of the imaginary part Π of one–loop corrections reads:

$$\begin{aligned}S&=\frac{-iD}{p^2-M^2}+\frac{-iD}{p^2-M^2}(T\Pi)\frac{-iD}{p^2-M^2}+\cdots\\&=\frac{-iD}{p^2-M^2+i\Pi}\left(\mathbb{1}+\frac{i\Pi}{p^2-M^2}(\mathbb{1}-T)\right).\end{aligned} \tag{25}$$

More explicitly

$$S^{\mu\nu}=\frac{-i}{p^2-M^2+i\Pi}\left\{g^{\mu\nu}-\frac{p^\mu p^\nu}{M^2}\left(1+\frac{i\Pi}{p^2}\right)\right\}. \tag{26}$$

Hence the introduction of a finite width for s-channel virtual W's which is required even in tree level calculations has to be associated with a corresponding modification of the $p^\mu p^\nu$ term.

3. Form factors for the vertex corrections

We report in this section the analytic expression of $Z^{\alpha\beta\mu}$ which is needed for actual computations in the FL scheme. Parametrizing $Z^{\alpha\beta\mu}$ as follows

$$\begin{aligned}Z^{\alpha\beta\mu}&=p_+^\alpha p_+^\beta p_+^\mu f_1+q^\alpha p_+^\beta p_+^\mu f_2+p_+^\alpha q^\beta p_+^\mu f_3\\&\quad+p_+^\alpha p_+^\beta q^\mu f_4+q^\alpha q^\beta p_+^\mu f_5+q^\alpha p_+^\beta q^\mu f_6\\&\quad+p_+^\alpha q^\beta q^\mu f_7+q^\alpha q^\beta q^\mu f_8+g^{\alpha\beta}p_+^\mu f_9\\&\quad+g^{\alpha\beta}q^\mu f_{10}+g^{\beta\mu}p_+^\alpha f_{11}+g^{\beta\mu}q^\alpha f_{12}\\&\quad+g^{\alpha\mu}p_+^\beta f_{13}+g^{\alpha\mu}q^\beta f_{14}\end{aligned} \tag{27}$$

we find

$$\begin{aligned}f_1=160\frac{p_+^2q^2p_-^2}{\lambda^3}\Bigg\{&f_0\Big[-6p_+^2q^2p_-^2-2(p_+^2+p_-^2)\\&\times(p_-\cdot p_+)^2+2(p_+^4+p_-^4)p_-\cdot p_+\Big]\\&+10\frac{p_+^2(p_-\cdot p_+)^2}{q^2}+20p_+^2p_-^2+10p_+^4\\&-\frac{2}{q^2}\left(3p_+^6+(p_-\cdot p_+)^2p_-^2+p_-^6\right)+2p_-^4\\&+\frac{\lambda}{10}\Bigg[f_0\Bigg(-\frac{q^2p_-^2}{p_+^2}+\frac{p_-^4}{p_+^2}-\frac{p_+^2q^2}{p_-^2}-6p_+^2\\&+\frac{p_+^4}{p_-^2}-10q^2-6p_-^2\Bigg)\frac{116}{3}\\&-2\frac{p_-^4}{q^2p_+^2}+2\frac{p_-^2}{p_+^2}+\frac{139}{3}\frac{p_+^2}{q^2}\\&+14\frac{p_+^2}{p_-^2}-\frac{20}{3}\frac{p_+^4}{q^2p_-^2}+\frac{67}{3}\frac{p_-^2}{q^2}\Bigg]\\&+\frac{\lambda^2}{10}\Bigg[-f_0\left(\frac{1}{p_+^2}+\frac{1}{p_-^2}\right)+\frac{7}{3p_+^2q^2}\\&+\frac{2}{3p_+^2p_-^2}+\frac{19}{3q^2p_-^2}\Bigg]\Bigg\},\end{aligned} \tag{28}$$

$$
\begin{aligned}
f_2 = & -160 \frac{p_+^2 q^2 p_-^2}{\lambda^3} \Bigg\{ f_0 \big[p_+^2 q^2 p_-^2 + p_+^2 q^4 \\
& - q^2 p_-^4 + 2 q^4 p_-^2 - q^6 \big] + 2 p_+^2 q \cdot p_- \\
& + 6 q^2 q \cdot p_- + 4 q^4 - 2 q \cdot p_- p_-^2 + \frac{f_0 \lambda}{20} \\
& \times \left(2 \frac{p_+^2 q^2}{p_-^2} + 3 p_+^2 + 17 q^2 - 2 \frac{q^4}{p_-^2} - 3 p_-^2 \right) \\
& + \frac{\lambda}{15 p_-^2} \left(11 q^2 + 13 q \cdot p_- - p_-^2 \right) \\
& + \frac{\lambda^2}{60 q^2 p_-^2} \left(1 + 3 f_0 q^2 \right) \Bigg\}, \qquad (29)
\end{aligned}
$$

$$
\begin{aligned}
f_3 = & 160 \frac{p_+^2 q^2 p_-^2}{\lambda^3} \Bigg\{ f_0 \big[3 p_+^2 q^2 p_-^2 \\
& + \left(3 p_+^2 - p_-^2 \right) (p_- \cdot p_+)^2 - 2 p_+^4 p_- \cdot p_+ \big] \\
& - 8 \frac{p_+^2 (p_- \cdot p_+)^2}{q^2} - 8 p_+^2 p_-^2 - 8 p_+^4 + 4 \frac{p_+^6}{q^2} \\
& + 4 \frac{(p_- \cdot p_+)^2 p_-^2}{q^2} \\
& + \frac{\lambda}{10} \Bigg[f_0 \left(\frac{p_+^2 q^2}{p_-^2} + 3 p_+^2 - \frac{p_+^4}{p_-^2} + \tfrac{13}{2} q^2 + 3 p_-^2 \right) \\
& - 20 - 32 \frac{p_+^2}{q^2} - \frac{40}{3} \frac{p_+^2}{p_-^2} \\
& + 6 \frac{p_+^4}{q^2 p_-^2} - 4 \frac{p_-^2}{q^2} \Bigg] \\
& + \frac{\lambda^2}{20} \Bigg[f_0 \left(\frac{1}{p_+^2} + \frac{2}{p_-^2} \right) \\
& - \frac{1}{p_+^2 q^2} - \frac{1}{p_+^2 p_-^2} - \frac{34}{3 q^2 p_-^2} \Bigg] \Bigg\}, \qquad (30)
\end{aligned}
$$

$$
\begin{aligned}
f_5 = & 160 \frac{p_+^2 q^2 p_-^2}{\lambda^3} \Bigg\{ f_0 \big[3 p_+^2 q^2 p_-^2 + p_+^2 q^4 - q^2 p_-^4 \\
& + 2 q^4 p_-^2 - q^6 \big] \\
& + 6 p_+^2 q \cdot p_- + 6 q^2 q \cdot p_- + 4 q^4 - 2 q \cdot p_- p_-^2 \\
& + \frac{f_0 \lambda}{20} \left(2 \frac{p_+^2 q^2}{p_-^2} + 7 p_+^2 + 21 q^2 - 2 \frac{q^4}{p_-^2} + p_-^2 \right) \\
& + \frac{\lambda}{5} \left(-3 + \frac{2}{3} \frac{q \cdot p_-}{q^2} + \frac{11}{3} \frac{q^2}{p_-^2} + 5 \frac{q \cdot p_-}{p_-^2} \right) \\
& + \frac{\lambda^2}{60 q^2 p_-^2} \left(1 + 3 f_0 q^2 \right) \Bigg\}, \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
f_9 = & 16 \frac{p_+^2 q^2 p_-^2}{\lambda^2} \Bigg\{ f_0 \left[q^2 p_- \cdot p_+ + \frac{\lambda}{2} \right] + 2 q \cdot p_+ \\
& + \frac{\lambda}{6 q^2 p_-^2} \left(4 p_+ \cdot q - 3 q^2 \right) \Bigg\} \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
f_{11} = & 16 \frac{p_+^2 q^2 p_-^2}{\lambda^2} \Bigg\{ f_0 \Bigg[q^2 p_- \cdot p_+ \\
& + \lambda \left(-\tfrac{1}{4} + \frac{p_- \cdot p_+}{2 p_+^2} \right) \Bigg] + 2 p_+ \cdot q \\
& + \lambda p_+ \cdot q \left(\frac{1}{2 p_+^2 q^2} + \frac{1}{2 p_+^2 p_-^2} - \frac{2}{3 q^2 p_-^2} \right) \Bigg\}, \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
f_{12} = & 16 \frac{p_+^2 q^2 p_-^2}{\lambda^2} \Bigg\{ f_0 \left[- p_+^2 q \cdot p_- + \frac{\lambda}{2} \right] \\
& - 2 p_+^2 + \frac{\lambda}{6 q^2 p_-^2} \left(6 q \cdot p_- - p_+^2 \right) \Bigg\}, \qquad (34)
\end{aligned}
$$

$$
\begin{aligned}
f_{13} = & 16 \frac{p_+^2 q^2 p_-^2}{\lambda^2} \Bigg\{ f_0 \Bigg[-7 q^2 p_+ \cdot q + 4 (p_+ \cdot q)^2 \\
& - 3 q^2 p_-^2 + 3 q^4 + \lambda \left(-\tfrac{3}{4} + \frac{p_+ \cdot q}{2 p_-^2} - \frac{q^2}{2 p_-^2} \right) \Bigg] \\
& + \tfrac{8}{3} \left(p_-^2 - q^2 \right) - \tfrac{22}{3} p_- \cdot p_+ + \frac{14}{3} \frac{(p_- \cdot p_+)^2}{p_-^2} \\
& + \frac{\lambda}{6 q^2 p_-^2} q \cdot p_- \Bigg\}, \qquad (35)
\end{aligned}
$$

$$f_{14} = 16\frac{p_+^2 q^2 p_-^2}{\lambda^2}\left\{ f_0\left[7p_+\cdot qq^2 - p_+\cdot qp_-^2 - 4(p_+\cdot q)^2 + 3q^2p_-^2 - 3q^4 + \lambda\left(\frac{1}{4} - \frac{p_+\cdot q}{2p_-^2} + \frac{q^2}{2p_-^2}\right)\right] - \frac{8}{3}\left(p_-^2 - q^2\right) + \frac{16}{3}p_-\cdot p_+ - \frac{14}{3}\frac{(p_-\cdot p_+)^2}{p_-^2} + \frac{\lambda}{6q^2p_-^2}\left(4p_-^2 - 5p_-\cdot p_+\right)\right\}, \tag{36}$$

with

$$f_0 = -\frac{2}{\sqrt{\lambda}}\ln\left(\frac{2(p_-\cdot q) + \sqrt{\lambda}}{2(p_-\cdot q) - \sqrt{\lambda}}\right),$$
$$\lambda \equiv 4(p_-\cdot q)^2 - 4p_-^2q^2. \tag{37}$$

The form factors f_4, f_6, f_7, f_8, f_{10} do not contribute in CC20 processes because the electron current is conserved. If we assume that all currents which couple to the fermion loops are conserved we have

$$Z^{\alpha\beta\mu} = q^\alpha q^\beta p_+^\mu c_0 + g^{\beta\mu}q^\alpha c_1 + g^{\alpha\mu}q^\beta c_2 + g^{\alpha\beta}p_+^\mu c_3, \tag{38}$$

$$c_0 = f_2 + f_5, \quad c_1 = f_{12}, \quad c_2 = f_{13} + f_{14},$$
$$c_3 = f_9, \tag{39}$$

where the c_i agree with the results of Ref. [2].

4. Applications to the process $e^-e^+\to e^-\bar{\nu}_e u\bar{d}$: numerical effects in a physically relevant case study

In the following we present numerical results obtained with the IFL scheme and make comparisons with those obtained with other gauge-preserving approaches. The following schemes are considered in our analysis:

Imaginary-part FL scheme (IFL): The imaginary part of the fermion-loop corrections Eq. (27)–(36) are used. The fermion masses are neglected in the loops but not in the rest of the diagrams.

Fixed width (FW): All W-boson propagators are given by

$$\frac{g^{\mu\nu} - \dfrac{p^\mu p^\nu}{M_W^2 - i\Gamma_W M_W}}{p^2 - M_W^2 + i\Gamma_W M_W}. \tag{40}$$

This gives an unphysical width for $p^2 < 0$, but retains U(1) gauge invariance.

Complex Mass (CM): All weak boson masses squared $M_B^2, B = W,Z$ are changed to $M_B^2 - i\gamma_B$, including when they appear in the definition of the weak mixing angle. This scheme has the advantage of preserving both U(1) and SU(2) Ward identities [18].

Overall scheme (OA): The diagrams for $e^-e^+\to e^-\bar{\nu}_e u\bar{d}$ can be split into two sets which are separately gauge invariant under U(1). In the present implementation of OA [19], t-channel diagrams are computed without any width and are then multiplied by $(q^2 - M^2)/(q^2 - M^2 + iM\Gamma)$ where q, M and Γ are the momentum, the mass and the width of the possibly-resonant W-boson. This scheme retains U(1) gauge invariance at the expenses of mistreating non resonant terms.

In order to assess the relevance of current non-conservation in process $e^-e^+\to e^-\bar{\nu}_e u\bar{d}$ we have also implemented the imaginary part of the fermion-loop corrections with the assumption that all currents which couple to the fermion-loop are conserved. In this case Eq. (27)–(36) reduce to those computed in [2]. Notice that the masses of external fermions are nonetheless taken into account in the calculation of the matrix elements. This scheme violates U(1) gauge invariance by terms which are proportional to the fermion masses squared, as already noted in Ref. [6]. However they are enhanced at high energy by large factors and can be numerically quite relevant. This scheme will be referred to as the imaginary-part FL scheme with conserved currents *(IFLCC)* in the following.

In the comparisons among the different codes mentioned in the introduction, `COMPHEP` and `WPHACT` used the OA scheme, `KORALW` and `GRC4F` the $L^{\mu\nu}$ transform method of Ref. [8], `NEXTCALIBUR` used the CM and `SWAP` the FW scheme. Here all schemes described above have been implemented in the new version of `WPHACT` in which all massive matrix elements have been added to the old massless ones. In particular the IFL contributions in Eq. (27)–(36) have been introduced. In this way, the same matrix elements, phase spaces and integration routines are used in all instances.

If not stated otherwise we apply the following cuts:

$$M(u\bar{d}) > 5\,\mathrm{GeV}, \quad E_u > 3\,\mathrm{GeV}, \quad E_{\bar{d}} > 3\,\mathrm{GeV},$$
$$\cos(\theta_e) > .997 \tag{41}$$

We have produced numerical results for $e^- e^+ \to e^- \bar{\nu}_e u\bar{d}$ in the small space-like q_γ^2 (collinear electron) region where we expect gauge-invariance issues to be essential. We have not included in our computations Initial State Radiation (ISR), in order to avoid any additional uncertainty in these comparisons among different gauge restoring schemes. In Table 1 we give the cross sections for CC20 at Lep 2 and LC energies. In Table 2 we give the cross sections for CC20 at $E = 800\,\mathrm{GeV}$ with slightly modified selections. With all other cuts at their standard values, in the second column the electron scattering angle is not allowed to be larger than 0.1 degree while in the third column the invariant mass of the $u\bar{d}$ pair is required to be greater than 40 GeV.

The IFL, FW, CM and OA schemes agree within $2\,\sigma$ in almost all cases. The IFLCC scheme agrees with all other ones at Lep 2 energies but already at 800 GeV it overestimates the total cross section by about 6%. At 1.5 TeV the error is almost a factor of two. The results in Table 2 show that the discrepancy between the IFLCC scheme and all the others decreases slightly to 5.6 % if larger masses of the $u\bar{d}$ pair are required. If instead smaller electron scattering angle are allowed the discrepancy increases to about 9%. This is a consequence of the fact that in the collinear region the neglected terms, proportional to the fermion masses, are enhanced by factors of order $\mathscr{O}\left(m_f^2 \gamma_W(p^2)/\left(M_W^2 m_e^2\right)\right)$ which can become very large at high energy even for typical light fermion masses.

We conclude then that, even in the presence of non–conserved currents i.e. of massive external fermions, the FW, CM and OA calculations give predictions which are in agreement, within a few per mil, with the IFL scheme. This agreement with the results of a fully self-consistent approach justifies from a practical point of view the ongoing use of the FW, CM and OA schemes. It should be remarked that for massless fermions it has been shown that at high energies, for the total cross section of the process $e^- e^+ \to \mu^- \bar{\nu}_\mu u\bar{d}$ the full FL scheme deviates from the FW scheme and the IFL scheme by about 2% at 1 TeV increasing to about 7% at 10 TeV [5] mainly because of the running of the couplings. As a consequence, it appears likely that calculations performed in the IFL scheme with running couplings would be able to reproduce the complete FL results with sufficient accuracy for most practical purposes. Hitherto missing higher order QCD and bosonic contributions could still conceivably produce significant corrections.

Table 1
Cross sections for the process $e^+ e^- \to e^- \bar{\nu}_e u\bar{d}$ for various gauge restoring schemes

	190 GeV	800 GeV	1500 GeV
IFL	0.11815 (13)	1.6978 (15)	3.0414 (35)
FW	0.11798 (11)	1.6948 (12)	3.0453 (41)
CM	0.11791 (12)	1.6953 (16)	3.0529 (60)
OA	0.11760 (10)	1.6953 (13)	3.0401 (23)
IFLCC	0.11813 (12)	1.7987 (16)	5.0706 (44)

Table 2
Cross sections for the process $e^+ e^- \to e^- \bar{\nu}_e u\bar{d}$ at E = 800 GeV for various gauge restoring schemes and different cuts

	$\cos(\theta_e) > .997$	$\theta_e < 0.1°$	$M(u\bar{d}) > 40$ GeV
IFL	1.6978 (15)	1.1550 (15)	1.6502 (15)
FW	1.6948 (12)	1.1538 (21)	1.6480 (13)
CM	1.6953 (16)	1.1533 (14)	1.6520 (10)
OA	1.6953 (13)	1.1537 (12)	1.6523 (12)
IFLCC	1.7987 (16)	1.2600 (22)	1.7424 (21)

5. Conclusions

The Imaginary Part Fermion-Loop scheme, introduced in Ref. [2] for the gauge-invariant treatment of the finite-width effects of W and Z bosons, has been generalized so that it could be applied to processes with massive external fermions. This involves the

Dyson resummation of higher order imaginary contributions to the propagator which implies, in the unitary gauge, a modification of the $p^{\mu}p^{\nu}$ term in the numerator. From a numerical point of view we find no significant difference between the IFL scheme and the FW, CM or OA schemes in the region most sensible to U(1) gauge invariance.

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